Contribution of Twist-3 Multi-Gluon Correlation Functions to Single Spin Asymmetry in Semi-Inclusive Deep Inelastic Scattering

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Abstract

As a possible source of the single transverse spin asymmetry, we study the contribution from purely gluonic correlation represented by the twist-3 "three-gluon correlation" functions in the transversely polarized nucleon. We first define a complete set of the relevant three-gluon correlation functions, and then derive its contribution to the twist-3 single-spin-dependent cross section for the D-meson production in semi-inclusive deep inelastic scattering, which is relevant to determine the three-gluon correlations. Our cross-section formula differs from the corresponding result in the literature, and the origin of the discrepancy is clarified.

1 Introduction

Clarifying the origin of the large single spin asymmetries (SSAs) observed in various high-energy semi-inclusive processes [1]-[7] has been a big challenge during the past decades. (See [8] for a review.) They are now understood as direct consequences of the orbital motion of quarks and gluons and/or the multi-parton correlations inside the hadrons, and thus provide a new opportunity for revealing the QCD dynamics and hadron structures, which do not appear in the parton models and perturbative QCD at the conventional twist-2 level. Satisfactory formulation of these effects in QCD, however, requires sophisticated framework for which lots of technical developments are involved. Up to now, the SSAs have been formulated based on the (naively) "T-odd" distribution/fragmentation functions [9]-[18] within the transverse-momentum-dependent (TMD) factorization approach [19, 20, 21], and on the twist-3 multi-parton correlation functions in the collinear factorization approach [22]-[38]. These two mechanisms, in principle, cover different kinematic regions, such that the former approach is suitable for treating SSAs in the region of the small transverse momentum of a particle observed in the final-state, while the latter is designed for systematic description of SSAs at large values of the corresponding transverse momentum. On the other hand, in the intermediate region of the corresponding transverse momentum where both approaches are valid, it's been shown for some structure functions in semi-inclusive deep inelastic scattering (SIDIS) and in the Drell-Yan cross section that the two approaches provide an equivalent description of SSAs [39, 40, 41, 42]. Therefore, in practice, the two approaches, tied together at the intermediate transverse-momentum region, can be regarded as providing a unique QCD effect leading to SSAs over the entire kinematic regions. Although our understanding on SSAs has made a great progress armed with these mechanisms, further studies are yet to be done for a complete clarification of all the effects responsible for SSAs. Among such effects, in particular, the role of purely gluonic effects has not been widely studied in the literature. Since the gluons are ample in the nucleon, they are potentially an important source of SSAs.

In this paper, we study a purely gluonic effect as an origin of SSAs in the framework of the collinear factorization. To this end, we work on SSAs for a heavy meson production in SIDIS, in particular, the D-meson production, $ep \to eDX$. Since this process is induced dominantly by the $c\bar{c}$-pair creation through photon-gluon fusion, this is the most relevant process to probe the gluonic effects for SSAs, together with D-meson production in pp collisions [43, 33] ongoing at RHIC [44]. In the collinear factorization framework, SSA is a twist-3 observable and thus the gluonic effects responsible for SSAs are represented by the twist-3 gluon correlation functions in the polarized nucleon, which were first introduced in the most general form by Ji [24]. Recently the authors of [32] studied the SSAs in SIDIS, $ep^\uparrow \to eDX$, applying the twist-3 mechanism. They derived a formula, in the leading order with respect to the QCD coupling constant, for the contribution to the single-spin-dependent cross section from a "three-gluon" correlation function. In our opinion, however, the starting formula used to derive the cross section in [32] is incomplete and cannot lead to the correct result. So we revisit the same issue in this paper.

As is well-known, a twist-2 gluon distribution inside the nucleon is defined in terms

of the gauge-invariant lightcone correlation function of the gluon field-strength tensors, schematically written as $\langle F_a^{\alpha+} F_a^{\beta+} \rangle$. Likewise, the twist-3 "three-gluon distribution" functions are defined through the gauge-invariant correlation functions $\langle C_{\pm}^{abc} F_a^{\alpha+} F_b^{\beta+} F_c^{\gamma+} \rangle$, with the structure constants of the color SU(3) group, $C_+^{abc} = if^{abc}$ and $C_-^{abc} = d^{abc}$. On the other hand, when we derive the three-gluon contribution to the cross section for $ep^\uparrow \to eDX$ with the large transverse momentum of the D meson, we start our analysis with a cut forward-amplitude for the cross section, in which the correlation functions of the gluon fields appear in the form $\sim \langle A_a^\alpha A_b^\beta A_c^\gamma \rangle$. Extraction of the twist-3 effect relevant for SSAs from the corresponding amplitude, converting eventually the associated nucleon matrix elements from $\langle A_a^\alpha A_b^\beta A_c^\gamma \rangle$ into the gauge-invariant forms $\langle C_{\pm}^{abc} F_a^{\alpha+} F_b^{\beta+} F_c^{\gamma+} \rangle$, is a highly nontrivial issue, unlike straightforward calculations of the twist-2 cross sections. In this connection, it is worth mentioning that, for the case where the twist-3 quark-gluon correlation functions participate, the necessary formulation was achieved in [27]; there, Ward identities played a crucial role to prove the factorization property and gauge invariance of the corresponding twist-3 cross section. Similarly, we will show that, owing to the constraints from the tree-level Ward identities satisfied by the relevant partonic hard parts, the twist-3 contribution to the cross section, associated with the three-gluon correlation functions $\langle A_a^\alpha A_b^\beta A_c^\gamma \rangle$, can be recast into the factorized expression in terms of the gauge-invariant functions $\langle C_{\pm}^{abc} F_a^{\alpha+} F_b^{\beta+} F_c^{\gamma+} \rangle$. With this formalism we will derive the complete single-spin-dependent cross section arising from the twist-3 three-gluon correlation functions of the nucleon. We will also clarify the difference of our result from that of [32].

The remainder of this paper is organized as follows: In section 2, we first define a complete set of the three-gluon correlation functions in the transversely polarized nucleon. We show that, actually, the three-gluon correlation functions defined in [24] are not all independent, i.e., some of them are redundant. So we newly define a genuine complete set of the three-gluon correlation functions. In section 3, we present our formalism for calculating the twist-3 single-spin-dependent cross section for $ep^\uparrow \to eDX$. We show that only a pole contribution of an internal propagator in the hard part leads to a real quantity relevant to the cross section, and that the tree-level Ward identities satisfied by the corresponding pole contributions play an essential role to give the factorized expression for the single-spin-dependent cross section, in terms of a complete set of the gauge-invariant correlation functions defined in section 2. In section 4, we present the final form of the single-spin-dependent cross section for $ep^\uparrow \to eDX$ and discuss its characteristic features. Section 5 is devoted to a brief summary of our result. In Appendix A, we summarize the relevant symmetry properties of the gluon correlation functions, which are used in section 3.

2 Three-gluon correlation functions in the transversely polarized nucleon

As a straightforward extension of the quark-gluon correlation functions discussed in, e.g., [26, 27], purely gluonic correlation functions can be defined as nucleon matrix elements of the three gluon fields on the lightcone [24]. Due to the different ways for contracting the

color indices of the three gluon fields to obtain the color-singlet operators, one can define the two-types of correlation functions as

$$O^{\alpha\beta\gamma}(x_1,x_2) = -gi^3 \int \frac{d\lambda}{2\pi} \int \frac{d\mu}{2\pi} e^{i\lambda x_1} e^{i\mu(x_2-x_1)} \langle pS| d^{bca} F_b^{\beta n}(0) F_c^{\gamma n}(\mu n) F_a^{\alpha n}(\lambda n)|pS\rangle, \quad (1)$$

$$N^{\alpha\beta\gamma}(x_1,x_2) = -gi^3 \int \frac{d\lambda}{2\pi} \int \frac{d\mu}{2\pi} e^{i\lambda x_1} e^{i\mu(x_2-x_1)} \langle pS| if^{bca} F_b^{\beta n}(0) F_c^{\gamma n}(\mu n) F_a^{\alpha n}(\lambda n)|pS\rangle, \quad (2)$$

where $F_a^{\alpha n} \equiv F_a^{\alpha\beta} n_\beta$ with $F_a^{\alpha\beta} = \partial^\alpha A_a^\beta - \partial^\beta A_a^\alpha + g f_{abc} A_b^\alpha A_c^\beta$ being the gluon field strength tensor, d^{bca} and f^{bca} are, respectively, the symmetric and anti-symmetric structure constants of the color SU(3) group, and we have suppressed the gauge-link operators which appropriately connect the field strength tensors so as to ensure the gauge invariance. p is the nucleon momentum, and S is the transverse spin vector of the nucleon normalized as $S^2 = -1$. We obtain the twist-3 contributions of (1) and (2), when we regard all the free Lorentz indices α, β, and γ to be transverse, and, in this twist-3 accuracy, p can be regarded as lightlike ($p^2 = 0$). n is another lightlike vector satisfying $p \cdot n = 1$, and, to be specific, we assume $p^\mu = (p^+, 0, \mathbf{0}_\perp)$ and $n^\mu = (0, n^-, \mathbf{0}_\perp)$; then, we have $S^\mu = (0, 0, \mathbf{S}_\perp)$.

Taking into account the constraints from hermiticity, invariance under the parity and time-reversal transformations, and the permutation symmetry among the participating three gluon-fields, Ji decomposed the twist-3 contribution of (1) in terms of the real, Lorentz-scalar functions O_1 and $\widetilde{O}_1$ associated with the six tensor structures as [24]

$$\begin{aligned} &2iM_N \Big[O_1(x_1,x_2) g^{\alpha\beta} \epsilon^{\gamma pnS} + O_1(x_2, x_2-x_1) g^{\beta\gamma} \epsilon^{\alpha pnS} + O_1(x_1, x_1-x_2) g^{\gamma\alpha} \epsilon^{\beta pnS} \\ &+ \widetilde{O}_1(x_1,x_2) \epsilon^{\alpha\beta pn} S^\gamma + \widetilde{O}_1(x_2, x_2-x_1) \epsilon^{\beta\gamma pn} S^\alpha - \widetilde{O}_1(x_1, x_1-x_2) \epsilon^{\gamma\alpha pn} S^\beta \Big], \end{aligned} \quad (3)$$

where we have introduced the nucleon mass M_N in order to define O_1 and $\widetilde{O}_1$ as dimensionless, and the similar decomposition of (2) was also introduced as [24]

$$\begin{aligned} &2iM_N \Big[N_1(x_1,x_2) g^{\alpha\beta} \epsilon^{\gamma pnS} - N_1(x_2, x_2-x_1) g^{\beta\gamma} \epsilon^{\alpha pnS} - N_1(x_1, x_1-x_2) g^{\gamma\alpha} \epsilon^{\beta pnS} \\ &+ \widetilde{N}_1(x_1,x_2) \epsilon^{\alpha\beta pn} S^\gamma - \widetilde{N}_1(x_2, x_2-x_1) \epsilon^{\beta\gamma pn} S^\alpha + \widetilde{N}_1(x_1, x_1-x_2) \epsilon^{\gamma\alpha pn} S^\beta \Big], \end{aligned} \quad (4)$$

with the other dimensionless, real functions N_1 and $\widetilde{N}_1$. In [24], the above four functions O_1, $\widetilde{O}_1$, N_1 and $\widetilde{N}_1$ were treated as independent twist-3 three-gluon correlation functions. However, the six tensor structures in (3) and (4) are not all independent and thus these decompositions actually define redundant correlation functions.[1] To see this, we recall the identity,

$$g^{\mu\nu}\epsilon^{\alpha\beta\rho\delta} = g^{\mu\alpha}\epsilon^{\nu\beta\rho\delta} + g^{\mu\beta}\epsilon^{\alpha\nu\rho\delta} + g^{\mu\rho}\epsilon^{\alpha\beta\nu\delta} + g^{\mu\delta}\epsilon^{\alpha\beta\rho\nu}, \quad (5)$$

and contract both sides of this identity with the tensor $g_\mu^\gamma S_\nu p_\rho n_\delta$. When the indices α, β and γ are associated with the transverse components, one obtains the relation,

$$\epsilon^{\alpha\beta pn} S^\gamma = -g^{\gamma\alpha}\epsilon^{\beta pnS} + g^{\beta\gamma}\epsilon^{\alpha pnS}, \quad (6)$$

[1] This point was also noticed and briefly mentioned in [54].

which shows that the tensor structures associated with $\widetilde{O}_1$ and $\widetilde{N}_1$ are reexpressed by those associated with O_1 and N_1, respectively, in (3) and (4). Accordingly, (1) and (2) should be decomposed into the three tensor structures. This can be formally achieved by omitting the terms associated with $\widetilde{O}_1$ and $\widetilde{N}_1$ in (3) and (4), respectively, and we newly define the two twist-3 functions $O(x_1, x_2)$ and $N(x_1, x_2)$ as independent three-gluon correlation functions to represent (1) and (2) as

$$\begin{aligned} &O^{\alpha\beta\gamma}(x_1, x_2) \\ &\quad = 2iM_N \left[O(x_1, x_2) g^{\alpha\beta}\epsilon^{\gamma pnS} + O(x_2, x_2 - x_1) g^{\beta\gamma}\epsilon^{\alpha pnS} + O(x_1, x_1 - x_2) g^{\gamma\alpha}\epsilon^{\beta pnS}\right], \end{aligned} \quad (7)$$

$$\begin{aligned} &N^{\alpha\beta\gamma}(x_1, x_2) \\ &\quad = 2iM_N \left[N(x_1, x_2) g^{\alpha\beta}\epsilon^{\gamma pnS} - N(x_2, x_2 - x_1) g^{\beta\gamma}\epsilon^{\alpha pnS} - N(x_1, x_1 - x_2) g^{\gamma\alpha}\epsilon^{\beta pnS}\right]. \end{aligned} \quad (8)$$

In this paper, we use these $O(x_1, x_2)$ and $N(x_1, x_2)$ as constituting a genuine complete set to express the contribution of the three-gluon correlations to the twist-3 single-spin-dependent cross section for $ep^\uparrow \to eDX$. Similarly to the decomposition (3) and (4) considered in [24], the independent tensor structures in (7) and (8) are parameterized by the common real functions O and N, respectively, as consequences of the constraints from hermiticity, invariance under the parity and time-reversal transformations, and the permutation symmetry among the participating three gluon-fields. In particular, these constraints imply the following symmetry relations for $O(x_1, x_2)$ and $N(x_1, x_2)$, which are associated with the C-odd and C-even combinations of the three gluon operators in (1) and (2), respectively:

$$O(x_1, x_2) = O(x_2, x_1), \qquad O(x_1, x_2) = O(-x_1, -x_2), \quad (9)$$

$$N(x_1, x_2) = N(x_2, x_1), \qquad N(x_1, x_2) = -N(-x_1, -x_2). \quad (10)$$

The authors of [45] also pointed out that there are only two independent three-gluon correlation functions at twist-3, in the study of the evolution equations for the twist-3 distributions in the transversely polarized nucleon. We also mention that the C-even three-gluon correlation function in (8) contributes to the twist-3 transverse-spin structure function $g_2(x, Q^2)$ in the inclusive DIS with polarized beam and target [46], and the twist-3 local operators associated with the double moments of (2) have been analyzed in the framework of the operator product expansion [47].

As we will see in the next section, a contribution to the single-spin-dependent cross section based on the corresponding factorization formula at the leading order in QCD perturbation theory is generated from a pole arising in the propagators in the hard partonic subprocesses, and such a pole contribution fixes the momentum fractions in the correlation functions $O^{\alpha\beta\gamma}(x_1, x_2)$ and $N^{\alpha\beta\gamma}(x_1, x_2)$ at $x_1 = x_2 \equiv x$. This represents the situation where one of the three gluon-lines participating in the partonic subprocesses has zero momentum (see (1), (2)), and thus the corresponding contributions relevant to SSA can be referred to as the soft-gluon-pole (SGP) contributions. Combined with the above decompositions (7) and

(8), the single-spin-dependent cross section proves to receive contributions associated with $O(x,x)$, $O(x,0)$, $N(x,x)$ and $N(x,0)$. In particular, we will observe in the next section that the partonic hard part associated with $O(x,x)$ is different from the hard part associated with $O(x,0)$, due to the difference in the tensor structures among the three terms in the right-hand side of (7). Similarly, the partonic hard part for $N(x,x)$ is different from that for $N(x,0)$, reflecting the difference in the corresponding tensor structures in (8).

Here, we comment on the treatment of the three-gluon correlation functions adopted in the recent work [32] on the same phenomenon as discussed in the present paper, i.e., on the twist-3 mechanism to SSAs in SIDIS, $ep^{\uparrow} \to eDX$. The three-gluon correlation functions introduced by the authors of [32] read

$$T_G^{(\pm)}(x,x) = \int \frac{dy_1^- dy_2^-}{2\pi} e^{ixp^+y_1^-} \frac{1}{xp^+} g_{\beta\alpha}\epsilon_{S\gamma np} \langle pS| C_{\pm}^{bca} F_b^{\beta +}(0) F_c^{\gamma +}(y_2^-) F_a^{\alpha +}(y_1^-)|pS\rangle, \quad (11)$$

with

$$C_{+}^{bca} = if^{bca}, \qquad C_{-}^{bca} = d^{bca}, \quad (12)$$

and it was claimed that the corresponding twist-3 single-spin-dependent cross section at the leading order in QCD perturbation theory can be expressed entirely in terms of these two functions of x (see (60) below and the discussion following this formula). However, $T_G^{(\pm)}(x,x)$ in (11) can be obtained by the contraction of $O^{\alpha\beta\gamma}(x,x)$ and $N^{\alpha\beta\gamma}(x,x)$ in (1) and (2) with the particular tensor $g_{\beta\alpha}\epsilon_{S\gamma np}$, and thus can be written, using (7) and (8), as

$$\frac{xg}{2\pi} T_G^{(+)}(x,x) = -4M_N \left(N(x,x) - N(x,0)\right), \quad (13)$$

$$\frac{xg}{2\pi} T_G^{(-)}(x,x) = -4M_N \left(O(x,x) + O(x,0)\right). \quad (14)$$

Using these relations, the twist-3 single-spin-dependent cross section obtained in [32] implies the same partonic hard parts for $O(x,x)$ and $O(x,0)$, and similarly for $N(x,x)$ and $N(x,0)$. Such result implied by [32] disagrees with our result as mentioned above: In the twist-3 single-spin-dependent cross section induced by the three-gluon correlations, $O(x,x)$ and $O(x,0)$ are associated with the different partonic hard parts, and similarly for $N(x,x)$ and $N(x,0)$, so that the corresponding cross section cannot be expressed by $T_G^{(\pm)}(x,x)$ only. In this connection, we also note that (7) and (8) can be converted to give $O(x,x)$, $O(x,0)$, $N(x,x)$ and $N(x,0)$ as

$$O(x,x) = \frac{i}{8M_N}\left(3g_{\alpha\beta}\epsilon_{\gamma pnS} - 2g_{\alpha\gamma}\epsilon_{\beta pnS}\right) O^{\alpha\beta\gamma}(x,x), \quad (15)$$

$$O(x,0) = \frac{-i}{8M_N}\left(g_{\alpha\beta}\epsilon_{\gamma pnS} - 2g_{\alpha\gamma}\epsilon_{\beta pnS}\right) O^{\alpha\beta\gamma}(x,x), \quad (16)$$

$$N(x,x) = \frac{i}{8M_N}\left(3g_{\alpha\beta}\epsilon_{\gamma pnS} - 2g_{\alpha\gamma}\epsilon_{\beta pnS}\right) N^{\alpha\beta\gamma}(x,x), \quad (17)$$

$$N(x,0) = \frac{i}{8M_N}\left(g_{\alpha\beta}\epsilon_{\gamma pnS} - 2g_{\alpha\gamma}\epsilon_{\beta pnS}\right) N^{\alpha\beta\gamma}(x,x). \quad (18)$$

From these relations, we see that the contributions of certain types of the twist-3 components in the three-gluon correlation functions, obtained as the contractions of $O^{\alpha\beta\gamma}(x,x)$, $N^{\alpha\beta\gamma}(x,x)$ with the tensor $g_{\alpha\gamma}\epsilon_{\beta pnS}$, were not taken into account in [32]. Taking into account the components contracted with $g_{\alpha\gamma}\epsilon_{\beta pnS}$, as well as those contracted with $g_{\alpha\beta}\epsilon_{\gamma pnS}$, is required on general grounds by the Bose statistics of the gluon, and, in (15)-(18), there is no reason to anticipate that the latter types of components are more important than the former.

3 Formalism for the twist-3 mechanism from three-gluon correlations

3.1 Kinematics for $ep^\uparrow \to eDX$

Here we summarize the kinematics for the SIDIS process,

$$e(\ell) + p^\uparrow(p,S) \to e(\ell') + D(P_h) + X. \tag{19}$$

As noted in the last section, one can assume that the initial nucleon's momentum p is lightlike, $p^2 = 0$, in the twist-3 accuracy. But, we keep the mass m_h of the final charmed hadron (D-meson) as $P_h^2 = m_h^2$. The corresponding results for the case of the light-meson production can be obtained by the replacement $m_h \to 0$ in all the formulae below. For the process (19), besides the masses of the participating particles, there are five independent Lorentz invariants as

$$S_{ep} = (p+\ell)^2, \quad x_{bj} = \frac{Q^2}{2p\cdot q}, \quad Q^2 = -q^2 = -(\ell-\ell')^2, \quad z_f = \frac{p\cdot P_h}{p\cdot q}, \quad q_T = \sqrt{-q_t^2}. \tag{20}$$

Here, q_t is the "transverse" component of q defined as

$$q_t^\mu = q^\mu + \left(\frac{m_h^2 p\cdot q}{(p\cdot P_h)^2} - \frac{P_h\cdot q}{p\cdot P_h}\right)p^\mu - \frac{p\cdot q}{p\cdot P_h}P_h^\mu, \tag{21}$$

satisfying $q_t\cdot p = q_t\cdot P_h = 0$. In the actual calculation we work in the "hadron frame" [48] where the virtual photon and the initial nucleon are collinear, i.e., both move along the z-axis. In this frame, specifically, their momenta q and p are given as

$$q^\mu = (q^0, \vec{q}) = (0,0,0,-Q)\ , \tag{22}$$

and, similarly,

$$p^\mu = \left(\frac{Q}{2x_{bj}}, 0, 0, \frac{Q}{2x_{bj}}\right)\ , \tag{23}$$

and the outgoing D-meson is assumed to reside in the xz plane:

$$P_h^\mu = \frac{z_f Q}{2}\left(1 + \frac{q_T^2}{Q^2} + \frac{m_h^2}{z_f^2 Q^2}, \frac{2q_T}{Q}, 0, -1 + \frac{q_T^2}{Q^2} + \frac{m_h^2}{z_f^2 Q^2}\right)\ . \tag{24}$$

The transverse momentum of the D-meson in this frame is given by $P_{hT} = z_f q_T$, which is true in any frame where the 3-momenta $\vec{q}$ and $\vec{p}$ are collinear. The lepton momentum in this frame can be parameterized as

$$\ell^\mu = \frac{Q}{2}\left(\cosh\psi, \sinh\psi\cos\phi, \sinh\psi\sin\phi, -1\right) ,$$
$$\ell'^\mu = \frac{Q}{2}\left(\cosh\psi, \sinh\psi\cos\phi, \sinh\psi\sin\phi, 1\right) , \tag{25}$$

where

$$\cosh\psi = \frac{2x_{bj}S_{ep}}{Q^2} - 1 . \tag{26}$$

We parameterize the transverse spin vector of the initial nucleon S^μ as

$$S^\mu = (0, \cos\Phi_S, \sin\Phi_S, 0), \tag{27}$$

where Φ_S represents the azimuthal angle of $\vec{S}$ measured from the hadron plane. With the above definition, the cross section for $ep^\uparrow \to eDX$ can be expressed in terms of S_{ep}, x_{bj}, Q^2, z_f, q_T^2, ϕ and Φ_S in the hadron frame. Note that ϕ and Φ_S are invariant under boosts in the $\vec{q}$-direction, so that the cross section presented below is the same in any frame where $\vec{q}$ and $\vec{p}$ are collinear.

3.2 Collinear expansion and gauge invariance for the three-gluon contribution

The differential cross section for $ep^\uparrow \to eDX$ can be obtained as

$$d\Delta\sigma = \frac{1}{2S_{ep}}\frac{d^3\vec{P}_h}{(2\pi)^3 2P_h^0}\frac{d^3\vec{\ell}'}{(2\pi)^3 2\ell'^0}\frac{e^4}{q^4}L_{\mu\nu}(\ell,\ell')W^{\mu\nu}(p,q,P_h), \tag{28}$$

where $L_{\mu\nu}(\ell,\ell') = 2(\ell_\mu\ell'_\nu + \ell_\nu\ell'_\mu) - Q^2 g_{\mu\nu}$ is the leptonic tensor for the unpolarized electron, and $W^{\mu\nu}(p,q,P_h)$ is the hadronic tensor. In the present study we are interested in the contribution to $W^{\mu\nu}(p,q,P_h)$ from the three-gluon correlation functions for the initial nucleon, in which c and $\bar{c}$ are created through the photon-gluon fusion process and one of them fragments into a D ($\bar{D}$) meson. The fragmentation function $D(z)$ for a c-quark to become the D-meson with momentum P_h is defined from the corresponding lightcone correlation function as

$$\sum_X \frac{1}{3}\int\frac{d\lambda}{2\pi}e^{-i\lambda/z}\langle 0|\psi_i(0)|D(P_h)X\rangle\langle D(P_h)X|\bar{\psi}_j(\lambda w)|0\rangle = (\not{p}_c + m_c)_{ij}\, D(z) + \cdots, \tag{29}$$

where the ellipses denote the terms associated with the gamma matrix structures which are irrelevant for the present purpose. In the left-hand side, we have suppressed the gauge-link

operators to be connected to the quark fields, as well as the trace over the color indices. Here, z is the relevant momentum fraction, w is the lightlike vector of $O(1/Q)$ satisfying $P_h \cdot w = 1$, and p_c is the momentum of the c (or $\bar{c}$) quark with mass m_c, such that $p_c^\mu = P_h^\mu/z + rw^\mu$, with $r = (m_c^2 z - m_h^2/z)/2$ to satisfy $p_c^2 = m_c^2$. At the leading twist-2 accuracy for the quark-fragmentation process, we set $w^\mu = p^\mu/(P_h \cdot p)$, and, in the hadron frame, p_c is expressed as

$$p_c^\mu = \frac{\hat{z}Q}{2}\left(1 + \frac{q_T^2}{Q^2} + \frac{m_c^2}{\hat{z}^2 Q^2}, \frac{2q_T}{Q}, 0, -1 + \frac{q_T^2}{Q^2} + \frac{m_c^2}{\hat{z}^2 Q^2}\right), \tag{30}$$

where $\hat{z} = \frac{z_f}{z}$. Note that, when we make the replacement, $\sum_X |D(P_h)X\rangle\langle D(P_h)X| \to |p_c'\rangle\langle p_c'|$ in the left-hand side of (29), with $|p_c'\rangle$ the state with a c-quark having the momentum $p_c' \equiv p_c|_{z\to z'}$, the ellipses in the right-hand side vanish and $D(z) \to \delta(1/z' - 1/z)$. The fragmentation function $D(z)$ of (29) is factorized from $W_{\mu\nu}$ as

$$W_{\mu\nu}(p, q, P_h) = \int \frac{dz}{z^2} D(z) w_{\mu\nu}\left(p, q, p_c\right), \tag{31}$$

where the summation over the c and $\bar{c}$ quark contributions is implicit. To extract the twist-3 effect in $w_{\mu\nu}$, one needs to analyze the diagrams of the type shown in Fig. 1. However, as shown in Appendix A, in the leading order with respect to the QCD coupling constant for the partonic hard scattering parts, the contribution of Fig. 1(a) can not give rise to the single-spin-dependent cross section, and only Fig. 1(b) contributes to SSA, due to the symmetry properties of the correlation functions of two and three gluon fields in the polarized nucleon. So we shall focus on the analysis of Fig. 1(b) below. The goal of our analysis is to show that all the contributions from Fig. 1(b) in the twist-3 accuracy can be expressed in terms of the gauge-invariant correlation functions $O^{\alpha\beta\gamma}(x_1, x_2)$ and $N^{\alpha\beta\gamma}(x_1, x_2)$ defined as (1) and (2) in the previous section. For this purpose, we work in Feynman gauge and apply the collinear expansion to the hard scattering part in Fig. 1(b), keeping all the terms contributing in the twist-3 accuracy [27]. For simplicity in the notation, we shall henceforth omit the Lorentz indices μ and ν for the virtual photon in $w_{\mu\nu}\left(p, q, p_c\right)$ of (31) and write it as $w\left(p, q, p_c\right)$.

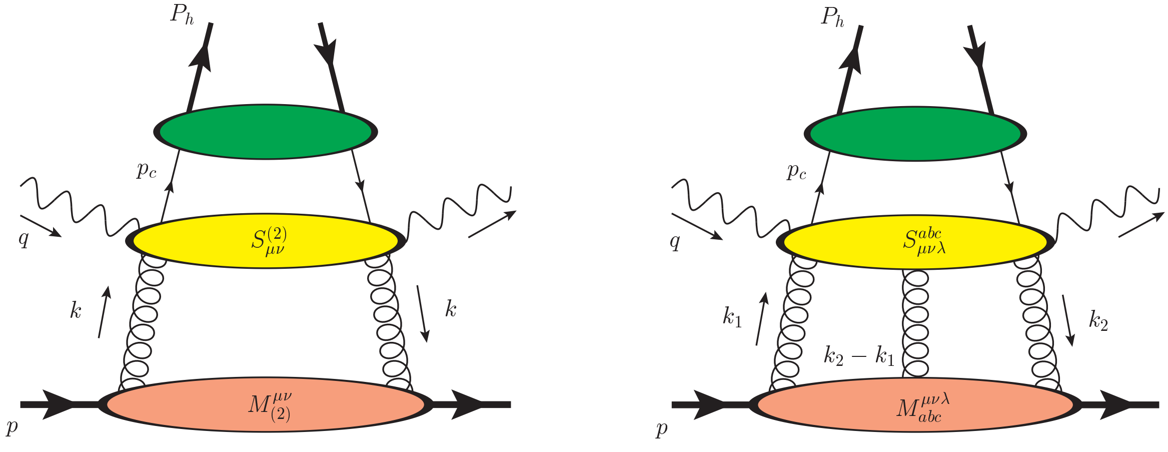
The contribution from Fig. 1(b) to $w(p, q, p_c)$ can be written as

$$w\left(p, q, p_c\right) = \int \frac{d^4k_1}{(2\pi)^4} \int \frac{d^4k_2}{(2\pi)^4} S_{\mu\nu\lambda}^{abc}\left(k_1, k_2, q, p_c\right) M_{abc}^{\mu\nu\lambda}(k_1, k_2), \tag{32}$$

where $S_{\mu\nu\lambda}^{abc}(k_1, k_2, q, p_c)$ is the partonic hard scattering part represented by the middle blob of Fig. 1(b) and $M_{abc}^{\mu\nu\lambda}(k_1, k_2)$ is the corresponding nucleon matrix element (lower blob) defined as

$$M_{abc}^{\mu\nu\lambda}(k_1, k_2) = g \int d^4\xi \int d^4\eta\, e^{ik_1\xi} e^{i(k_2-k_1)\eta} \langle pS| A_b^\nu(0) A_c^\lambda(\eta) A_a^\mu(\xi) |pS\rangle. \tag{33}$$

Note that, for later convenience, we include one QCD coupling constant in the definition of this nucleon matrix element. In (32), a real contribution relevant to the cross section for



(a) (b)

Figure 1: Generic diagrams for the hadronic tensor of $ep^{\uparrow} \to eDX$ induced by the gluonic effect in the nucleon. Each one is decomposed into the nucleon matrix element (lower blob), D-meson matrix element (upper blob), and the partonic hard scattering part by the virtual photon (middle blob). In the expansion by the number of gluon lines connecting the middle and lower blobs, the first two terms, (a) and (b), are relevant to the twist-3 effect induced by the gluons in the nucleon.

SSA occurs from an imaginary part of the color-projected hard part $C^{bca}_{\pm} S^{abc}_{\mu\nu\lambda}(k_1, k_2, q, p_c)$ with (12), since $C^{bca}_{\pm} M^{\mu\nu\lambda}_{abc}(k_1, k_2)$ are pure imaginary quantities as shown in Appendix A. This means that only the pole contribution produced by an internal propagator in the hard part can give rise to SSA.

In the leading order with respect to the QCD coupling constant, we find that four topologically distinct diagrams shown in Fig. 2, together with their mirror diagrams, give rise to the "surviving" pole contributions; here, a short bar indicates the quark propagator that produces the corresponding pole contribution. The other pole contributions turn out to cancel among themselves after summing the contributions of all the leading-order diagrams for (32). With the assignment of the momenta k_1 and k_2 of gluons as shown in Fig. 2, the condition for those poles is given by $(p_c - k_2 + k_1)^2 - m_c^2 = 0$. After we perform the collinear expansion and reach the collinear limit, $k_i \to x_i p$ ($i = 1,\ 2$), with x_1, x_2 and $x_2 - x_1$ representing the longitudinal momentum fractions of the relevant three gluons, this condition reduces to $x_1 = x_2$ and hence a pole of such type is referred to as the soft-gluon pole (SGP). In the following, we assume that $S^{abc}_{\mu\nu\lambda}(k_1, k_2, q, p_c)$ in (32) represents the sum of the contributions of the diagrams in Fig. 2 and their mirror diagrams, in which the barred propagator is replaced by its pole contribution. Also, for simplicity of notation, we suppress the color indices a, b, c and the momenta q and p_c in the hard part $S^{abc}_{\mu\nu\lambda}(k_1, k_2, q, p_c)$, writing it simply as $S_{\mu\nu\lambda}(k_1, k_2)$, and correspondingly, we write $M^{\mu\nu\lambda}_{abc}(k_1, k_2)$ as $M^{\mu\nu\lambda}(k_1, k_2)$.

To perform the collinear expansion, we decompose the relevant gluon momenta k_i ($i =$

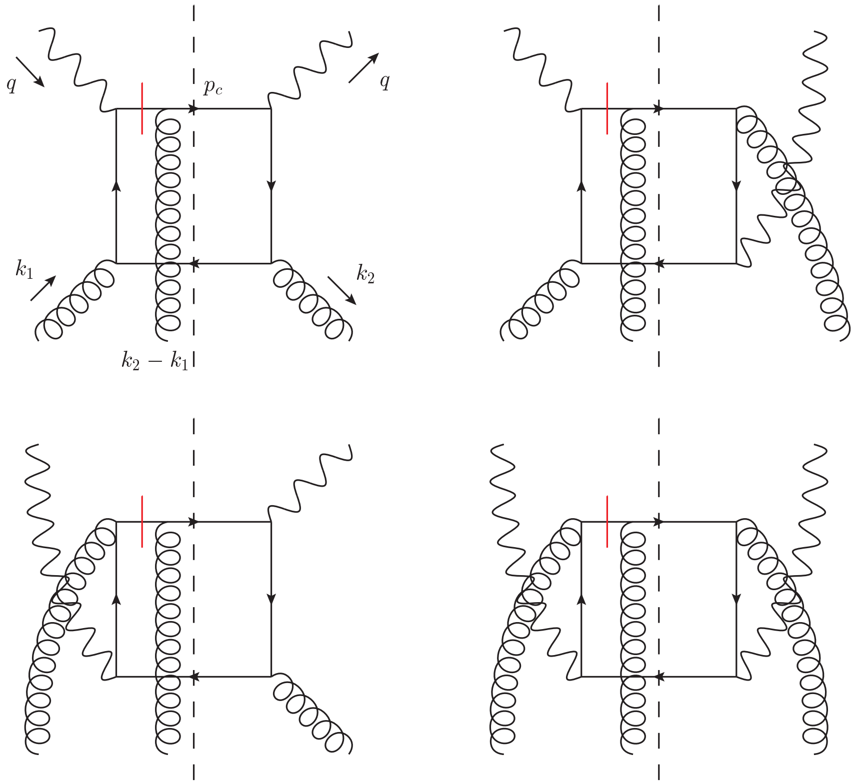


Figure 2: Feynman diagrams for the partonic hard part in Fig. 1(b), representing the photon-gluon fusion subprocesses that give rise to the "surviving" pole contribution for $ep^{\uparrow} \to eDX$ in the leading order with respect to the QCD coupling constant. The short bar on the internal c-quark line indicates that the pole part is to be taken from that propagator. In the text, momenta are assigned as shown in the upper-left diagram, where p_c denotes the momentum of the c-quark fragmenting into the D-meson in the final state. The mirror diagrams also contribute.

$1, 2)$ as

$$k_i^{\mu} = (k_i \cdot n)p^{\mu} + (k_i \cdot p)n^{\mu} + k_{\perp}^{\mu} \equiv x_i p^{\mu} + \omega^{\mu}{}_{\nu} k_i^{\nu}, \tag{34}$$

where $x_i = k_i \cdot n$ and $\omega^{\mu}{}_{\nu} \equiv g^{\mu}{}_{\nu} - p^{\mu} n_{\nu}$. Since $p^{\mu} \sim g_{+}^{\mu} Q$ in the hadron frame with (23) and thus the component along p^{μ} gives the leading contribution in (34) with respect to the hard scale Q, we expand $S_{\mu\nu\lambda}(k_1, k_2)$ around $k_i = x_i p$. Expressing also the gluon field A^{α} in the Feynman gauge as

$$A^{\alpha} = (p^{\alpha} n_{\kappa} + \omega^{\alpha}{}_{\kappa})\, A^{\kappa} = p^{\alpha} n \cdot A + \omega^{\alpha}{}_{\kappa} A^{\kappa}, \tag{35}$$

we note that, in the matrix element $M^{\mu\nu\lambda}(k_1, k_2)$ of (33), the components associated with the second term of (35) give rise to the contributions suppressed by $\sim 1/Q$ or more, compared

with the corresponding contribution due to the first term, $p^\alpha n\cdot A$ (see, e.g., [27]). According to the decomposition (35), the integrand of (32) can be expressed as

$$
\begin{aligned}
&S_{\mu\nu\lambda}(k_1,k_2)\,M^{\mu\nu\lambda}(k_1,k_2)\\
&\quad= S_{\mu\nu\lambda}(k_1,k_2)\,(p^\mu n_\kappa+\omega^\mu{}_\kappa)(p^\nu n_\tau+\omega^\nu{}_\tau)(p^\lambda n_\sigma+\omega^\lambda{}_\sigma)M^{\kappa\tau\sigma}(k_1,k_2)\\
&\quad= S_{ppp}(k_1,k_2)\,M^{nnn}(k_1,k_2)+S_{\alpha pp}(k_1,k_2)\,\omega^\alpha{}_\kappa M^{\kappa nn}(k_1,k_2)+S_{p\alpha p}(k_1,k_2)\,\omega^\alpha{}_\kappa M^{n\kappa n}(k_1,k_2)\\
&\quad+\cdots+S_{p\alpha\beta}(k_1,k_2)\,\omega^\alpha{}_\kappa\omega^\beta{}_\tau M^{n\kappa\tau}(k_1,k_2)+S_{\alpha\beta\gamma}(k_1,k_2)\,\omega^\alpha{}_\kappa\omega^\beta{}_\tau\omega^\gamma{}_\sigma M^{\kappa\tau\sigma}(k_1,k_2),
\end{aligned}
\tag{36}
$$

where $S_{ppp}(k_1,k_2)\equiv S_{\mu\nu\lambda}(k_1,k_2)p^\mu p^\nu p^\lambda$, $M^{nnn}(k_1,k_2)\equiv M^{\mu\nu\lambda}(k_1,k_2)n_\mu n_\nu n_\lambda$, etc. By performing the collinear expansion of the hard part of each term in the right-hand side of this formula, we can organize the integrand of (32) based on the order counting with (34), (35), keeping the terms necessary in the twist-3 accuracy. For the first term in the right-hand side of (36), the Taylor expansion about $k_i=x_ip$ gives,

$$
\begin{aligned}
&S_{ppp}(k_1,k_2)\\
&\quad= S_{ppp}(x_1,x_2)+\omega^\alpha{}_\kappa k_1^\kappa\,\frac{\partial S_{ppp}(k_1,k_2)}{\partial k_1^\alpha}\bigg|_{k_i=x_ip}+\omega^\alpha{}_\kappa k_2^\kappa\,\frac{\partial S_{ppp}(k_1,k_2)}{\partial k_2^\alpha}\bigg|_{k_i=x_ip}\\
&\quad+\frac{1}{2}\omega^\alpha{}_\kappa k_1^\kappa\,\omega^\beta{}_\tau k_1^\tau\,\frac{\partial^2 S_{ppp}(k_1,k_2)}{\partial k_1^\alpha\partial k_1^\beta}\bigg|_{k_i=x_ip}+\frac{1}{2}\omega^\alpha{}_\kappa k_2^\kappa\,\omega^\beta{}_\tau k_2^\tau\,\frac{\partial^2 S_{ppp}(k_1,k_2)}{\partial k_2^\alpha\partial k_2^\beta}\bigg|_{k_i=x_ip}\\
&\quad+\omega^\alpha{}_\kappa k_1^\kappa\,\omega^\beta{}_\tau k_2^\tau\,\frac{\partial^2 S_{ppp}(k_1,k_2)}{\partial k_1^\alpha\partial k_2^\beta}\bigg|_{k_i=x_ip}\\
&\quad+\frac{1}{6}\omega^\alpha{}_\kappa k_1^\kappa\,\omega^\beta{}_\tau k_1^\tau\,\omega^\gamma{}_\sigma k_1^\sigma\,\frac{\partial^3 S_{ppp}(k_1,k_2)}{\partial k_1^\alpha\partial k_1^\beta\partial k_1^\gamma}\bigg|_{k_i=x_ip}+\frac{1}{6}\omega^\alpha{}_\kappa k_2^\kappa\,\omega^\beta{}_\tau k_2^\tau\,\omega^\gamma{}_\sigma k_2^\sigma\,\frac{\partial^3 S_{ppp}(k_1,k_2)}{\partial k_2^\alpha\partial k_2^\beta\partial k_2^\gamma}\bigg|_{k_i=x_ip}\\
&\quad+\frac{1}{2}\omega^\alpha{}_\kappa k_1^\kappa\,\omega^\beta{}_\tau k_1^\tau\,\omega^\gamma{}_\sigma k_2^\sigma\,\frac{\partial^3 S_{ppp}(k_1,k_2)}{\partial k_1^\alpha\partial k_1^\beta\partial k_2^\gamma}\bigg|_{k_i=x_ip}+\frac{1}{2}\omega^\alpha{}_\kappa k_1^\kappa\,\omega^\beta{}_\tau k_2^\tau\,\omega^\gamma{}_\sigma k_2^\sigma\,\frac{\partial^3 S_{ppp}(k_1,k_2)}{\partial k_1^\alpha\partial k_2^\beta\partial k_2^\gamma}\bigg|_{k_i=x_ip}\\
&\quad+\cdots.
\end{aligned}
\tag{37}
$$

Here and below, we use the notation $S_{\mu\nu\lambda}(x_1,x_2)\equiv S_{\mu\nu\lambda}(x_1p,x_2p)$ for simplicity. We have written down the expansion explicitly up to the third-order terms. This is because, according to the above notice with the decompositions (34), (35), the third-order terms in (37) behave as the same order as the first term in

$$
S_{\alpha\beta\gamma}(k_1,k_2)\omega^\alpha{}_\kappa\omega^\beta{}_\tau\omega^\gamma{}_\sigma M^{\kappa\tau\sigma}(k_1,k_2)=\left[S_{\alpha\beta\gamma}(x_1,x_2)+\cdots\right]\omega^\alpha{}_\kappa\omega^\beta{}_\tau\omega^\gamma{}_\sigma M^{\kappa\tau\sigma}(k_1,k_2),
\tag{38}
$$

which is obtained by the Taylor expansion of the last term of (36). If we substitute (38) directly into the integrand of (32) and perform the integrals over k_1 and k_2, the first term

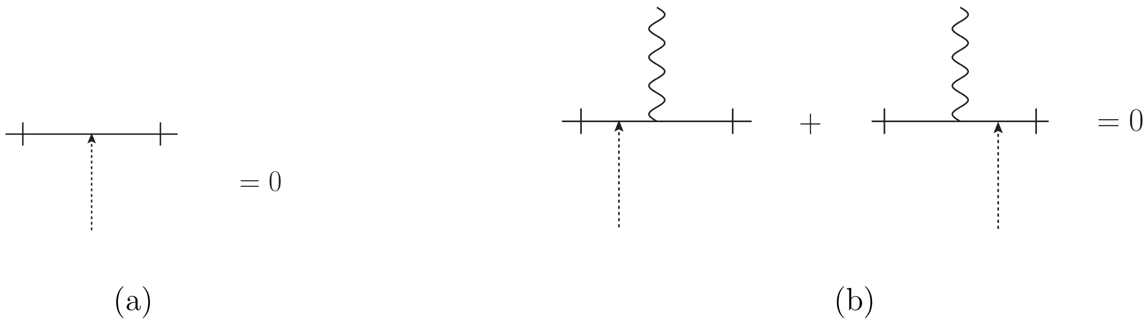


Figure 3: Ward identities used for the pole contribution. The dotted line represents a scalar-polarized gluon, and the quark lines marked by a bar are on-shell.

in the right-hand side produces the contribution, which is associated with the hard scattering between the three physical gluons from the nucleon and behaves as the same order as the formal convolution of $S_{\alpha\beta\gamma}(x_1, x_2)$ with the twist-3 correlation functions of (1) and (2). Actually, this corresponds to a quantity of twist-3. Compared to this, the ellipses in (38) give rise to the terms suppressed by $1/Q$ or more corresponding to twist-4 and higher, and thus are irrelevant here. We can write down the collinear expansions for the hard parts associated with the other terms in (36), similarly as (37): We expand $S_{\alpha pp}(k_1, k_2)$, $S_{p\alpha p}(k_1, k_2)$ and $S_{pp\alpha}(k_1, k_2)$ through the terms involving the second derivatives, and $S_{\alpha\beta p}(k_1, k_2)$, $S_{\alpha p\beta}(k_1, k_2)$ and $S_{p\alpha\beta}(k_1, k_2)$ through the terms involving the first derivatives. Thus, the collinear expansion of $S_{\mu\nu\lambda}(k_1, k_2)$ in (32) produces lots of terms as above, and each of those terms is not gauge invariant. At first sight, it looks hopeless to reorganize those into a form of the convolution with only the gauge-invariant correlation functions $O^{\alpha\beta\gamma}(x_1, x_2)$ and $N^{\alpha\beta\gamma}(x_1, x_2)$ of (1) and (2) used.

However, as was the case for the pion production associated with the twist-3 quark-gluon correlation functions [27], great simplification occurs due to Ward identities satisfied by the corresponding partonic hard-scattering function, $S_{\mu\nu\lambda}(k_1, k_2)$. To derive the Ward identities, we note that the contribution to $S_{\mu\nu\lambda}(k_1, k_2)$ from each diagram in Fig. 2 all contains the two delta functions, $\delta\left((k_2 + q - p_c)^2 - m_c^2\right)$ and $\delta\left((p_c + k_1 - k_2)^2 - m_c^2\right)$, representing the on-shell conditions associated, respectively, with the final-state cut on the unobserved $\bar{c}$-quark line and with the unpinched pole contribution of the barred propagator. Also, in each diagram in Fig. 2, the c-quark line fragmenting into the final-state D-meson is on-shell, see (30). Due to these on-shell conditions for the diagrams in Fig. 2 and the similar conditions for their mirror diagrams, $S_{\mu\nu\lambda}(k_1, k_2)$ satisfies the tree-level Ward identities,

$$
\begin{aligned}
&k_1^\mu S_{\mu\nu\lambda}(k_1, k_2) = 0, \\
&k_2^\nu S_{\mu\nu\lambda}(k_1, k_2) = 0, \\
&(k_2 - k_1)^\lambda S_{\mu\nu\lambda}(k_1, k_2) = 0.
\end{aligned}
\tag{39}
$$

Here, the last identity and the first two identities are represented diagrammatically in

Figs. 3(a) and 3(b), respectively. In the collinear limit, $k_i \to x_i p$ $(i = 1,\ 2)$, we have $\delta\left((p_c + k_1 - k_2)^2 - m_c^2\right) \to (1/2p_c \cdot p)\delta(x_1 - x_2)$, while the other delta function associated with the diagrams in Fig. 2 implies $x_2 > x_{bj}$ (see (59) below); the similar relations hold also for the corresponding mirror diagrams. Thus, the collinear expansion of Ward identities of (39) produces a series of relations in the collinear limit:

$$S_{p\nu\lambda}(x_1, x_2) = 0, \tag{40}$$

$$S_{\mu p\lambda}(x_1, x_2) = 0, \tag{41}$$

$$(x_2 - x_1)S_{\mu\nu p}(x_1, x_2) = 0, \tag{42}$$

$$\left.\frac{\partial S_{\mu p\lambda}(k_1, k_2)}{\partial k_1^\alpha}\right|_{k_i = x_i p} = 0, \qquad \left.\frac{\partial S_{p\nu\lambda}(k_1, k_2)}{\partial k_2^\alpha}\right|_{k_i = x_i p} = 0, \tag{43}$$

$$S_{\alpha\nu\lambda}(x_1, x_2) + x_1 \left.\frac{\partial S_{p\nu\lambda}(k_1, k_2)}{\partial k_1^\alpha}\right|_{k_i = x_i p} = 0, \tag{44}$$

$$S_{\mu\beta\lambda}(x_1, x_2) + x_2 \left.\frac{\partial S_{\mu p\lambda}(k_1, k_2)}{\partial k_2^\beta}\right|_{k_i = x_i p} = 0, \tag{45}$$

$$\left.\frac{\partial^2 S_{\mu p\lambda}(k_1, k_2)}{\partial k_1^\alpha \partial k_1^\beta}\right|_{k_i = x_i p} = \left.\frac{\partial^2 S_{p\nu\lambda}(k_1, k_2)}{\partial k_2^\alpha \partial k_2^\beta}\right|_{k_i = x_i p} = 0, \tag{46}$$

$$x_2 \left.\frac{\partial^2 S_{\mu p\lambda}(k_1, k_2)}{\partial k_1^\alpha \partial k_2^\beta}\right|_{k_i = x_i p} + \left.\frac{\partial S_{\mu\beta\lambda}(k_1, k_2)}{\partial k_1^\alpha}\right|_{k_i = x_i p} = 0, \tag{47}$$

and so on, where $\alpha \neq +$, $\beta \neq +$. These can be used to reorganize various terms obtained by the collinear expansion of (36), and one obtains the following results for the contribution in each order:

(i) By the relations (40), (41) and (43), the first three terms in (37), and all terms arising in (36) up to the order in $1/Q$ of those three terms, vanish.

(ii) By the relations (42), (44)-(47), the sum of the contributions of the next higher order in (36), which behave as the same order as the terms involving the second derivative in (37), vanish (see the discussion below (52)).

(iii) The remaining contributions in (36), behaving as the same order as the first term in (38), eventually yield the gauge-invariant twist-3 contribution to $w(p, q, p_c)$ in (57) below.

Instead of describing in detail those rather lengthy calculations in the framework of the standard collinear expansion, we may employ a somewhat different approach: Among

the relevant relations (40)-(47), in particular, (44), (45) and (47) play a role to connect the two terms that are generated from the different terms in the right-hand side of (36) through the collinear expansion. Namely, to obtain the gauge-invariant result, we have to combine the contributions from different terms in (36), and, furthermore, those terms are associated with different numbers of derivatives. These facts suggest an approach to apply directly Ward identities of (39) to the second line of (36), before the collinear expansion, i.e., without the expansion implied by the second equality in (36), nor the Taylor expansion about $k_i = x_i p$. Indeed, substituting the decomposition (34) into the first two identities in (39), we obtain

$$
\begin{aligned}
S_{p\nu\lambda}(k_1,k_2) &= -\frac{1}{x_1}\omega^{\mu}{}_{\alpha}k_1^{\alpha}S_{\mu\nu\lambda}(k_1,k_2),\\
S_{\mu p\lambda}(k_1,k_2) &= -\frac{1}{x_2}\omega^{\nu}{}_{\beta}k_2^{\beta}S_{\mu\nu\lambda}(k_1,k_2),
\end{aligned}
\tag{48}
$$

and we apply these relations to the second line of (36), yielding

$$
\begin{aligned}
&S_{\mu\nu\lambda}\left(k_1,k_2\right)M^{\mu\nu\lambda}(k_1,k_2)\\
&= S_{\mu\nu\lambda}\left(k_1,k_2\right)\frac{1}{x_1}\omega^{\mu}{}_{\alpha}\left(-k_1^{\alpha}n_{\kappa}+k_1\cdot n g^{\alpha}_{\kappa}\right)\\
&\quad\times\frac{1}{x_2}\omega^{\nu}{}_{\beta}\left(-k_2^{\beta}n_{\tau}+k_2\cdot n g^{\beta}_{\tau}\right)\left(p^{\lambda}n_{\sigma}+\omega^{\lambda}{}_{\sigma}\right)M^{\kappa\tau\sigma}(k_1,k_2),
\end{aligned}
\tag{49}
$$

where some factors combined with the matrix element (33) in the right-hand side can be reexpressed as, restoring the color indices a, b and c,

$$
\begin{aligned}
\left(-k_1^{\alpha}n_{\kappa}+k_1\cdot n g^{\alpha}_{\kappa}\right)\left(-k_2^{\beta}n_{\tau}+k_2\cdot n g^{\beta}_{\tau}\right)M^{\kappa\tau\sigma}_{abc}(k_1,k_2) &\equiv M^{\alpha\beta\sigma}_{A,abc}(k_1,k_2)\\
&= g\int d^4\xi\int d^4\eta\, e^{ik_1\xi}e^{i(k_2-k_1)\eta}\langle pS|F_b^{\beta n}(0)A_c^{\sigma}(\eta)F_a^{\alpha n}(\xi)|pS\rangle,
\end{aligned}
\tag{50}
$$

up to the correction terms beyond the present lowest-order calculation in QCD perturbation theory, so that

$$
S_{\mu\nu\lambda}\left(k_1,k_2\right)M^{\mu\nu\lambda}(k_1,k_2) = S_{\mu\nu\lambda}\left(k_1,k_2\right)\frac{\omega^{\mu}{}_{\alpha}\omega^{\nu}{}_{\beta}}{x_1x_2}\left[p^{\lambda}M_A^{\alpha\beta n}(k_1,k_2)+\omega^{\lambda}{}_{\sigma}M_A^{\alpha\beta\sigma}(k_1,k_2)\right].
\tag{51}
$$

As the next step, we perform the collinear expansion of the hard-scattering function. Based on the definition (50) and the decomposition (35), we note that the first and second terms in the parentheses in (51) correspond, respectively, to the "second" and "third" orders in the order counting relevant to the collinear expansion like (37). Thus, the collinear expansion of (51), up to the desired order, reads (see the discussion below (38))

$$
S_{\mu\nu\lambda}\left(k_1,k_2\right)M^{\mu\nu\lambda}(k_1,k_2) = \frac{\omega^{\mu}{}_{\alpha}\omega^{\nu}{}_{\beta}}{x_1x_2}\left[\left(S_{\mu\nu p}\left(x_1,x_2\right)+\omega^{\lambda}{}_{\kappa}k_1^{\kappa}\left.\frac{\partial S_{\mu\nu p}(k_1,k_2)}{\partial k_1^{\lambda}}\right|_{k_i=x_ip}\right.\right.
$$

$$+\omega^{\lambda}{}_{\kappa}k_2^{\kappa}\left.\frac{\partial S_{\mu\nu p}(k_1,k_2)}{\partial k_2^{\lambda}}\right|_{k_i=x_ip}\Bigg)M_A^{\alpha\beta n}(k_1,k_2)+S_{\mu\nu\lambda}(x_1,x_2)\,\omega^{\lambda}{}_{\sigma}M_A^{\alpha\beta\sigma}(k_1,k_2)\Bigg] \quad (52)$$

First, we consider the first term in the right-hand side, which is proportional to $S_{\mu\nu p}(x_1,x_2) = S_{\mu\nu\lambda}(x_1,x_2)\,p^{\lambda}$ and corresponds to the second-order term in the above-mentioned order counting. One can show that, by the direct diagrammatic calculation of $S_{\mu\nu p}(x_1,x_2)$, the corresponding SGP contributions from the diagrams in Fig. 2 cancel with those from their mirror diagrams (i.e., $S_{\mu\nu p}(x_1,x_2)$ arising in (42) equals zero even for $x_1 = x_2$), and thus the first term in the parentheses $(\cdots)$ in the right-hand side of (52) vanishes. It is worth noting that the similar vanishing property of the SGP contributions was used also for the case of the pion production associated with the twist-3 quark-gluon correlation functions (see, e.g., [27]). On the other hand, $S_{\mu\nu\lambda}(x_1,x_2)\,\omega^{\lambda}{}_{\sigma}$, arising in the last term in (52), does not vanish, but this can be recast using the relation,

$$S_{\mu\nu\lambda}(x_1,x_2)\,\omega^{\lambda}{}_{\sigma} = (x_1-x_2)\,\omega^{\lambda}{}_{\sigma}\left.\frac{\partial S_{\mu\nu p}(k_1,k_2)}{\partial k_2^{\lambda}}\right|_{k_i=x_ip}, \quad (53)$$

which is obtained by the collinear expansion of the last Ward identity of (39). Furthermore, for the second term in the right-hand side of (52), we may use another relation,

$$\left.\frac{\partial S_{\mu\nu p}(k_1,k_2)}{\partial k_1^{\lambda}}\right|_{k_i=x_ip} = -\left.\frac{\partial S_{\mu\nu p}(k_1,k_2)}{\partial k_2^{\lambda}}\right|_{k_i=x_ip}, \quad (54)$$

which can be derived for $\lambda = \perp$ by direct inspection of the diagrams in Fig. 2 and their mirror diagrams. We remind that a similar relation also holds for the hard part corresponding to the quark-gluon correlation functions, as discussed for the case of the pion production [27]. Thus, (52) yields, at the twist-3 accuracy,

$$\begin{aligned}S_{\mu\nu\lambda}(k_1,k_2)\,M^{\mu\nu\lambda}(k_1,k_2) = {}&\frac{\omega^{\mu}{}_{\alpha}\omega^{\nu}{}_{\beta}}{x_1x_2}\omega^{\lambda}{}_{\kappa}\left.\frac{\partial S_{\mu\nu p}(k_1,k_2)}{\partial k_2^{\lambda}}\right|_{k_i=x_ip}\\ &\times\left[-\left(k_1^{\kappa}-k_2^{\kappa}\right)n_{\sigma}+(k_1-k_2)\cdot n g_{\sigma}^{\kappa}\right]M_A^{\alpha\beta\sigma}(k_1,k_2),\end{aligned} \quad (55)$$

where the second line gives, similarly as in (50),

$$\begin{aligned}&\left[-\left(k_1^{\kappa}-k_2^{\kappa}\right)n_{\sigma}+(k_1-k_2)\cdot n g_{\sigma}^{\kappa}\right]M_{A,abc}^{\alpha\beta\sigma}(k_1,k_2)\\ &\quad = ig\int d^4\xi\int d^4\eta\, e^{ik_1\xi}e^{i(k_2-k_1)\eta}\langle pS|F_b^{\beta n}(0)F_c^{\kappa n}(\eta)F_a^{\alpha n}(\xi)|pS\rangle,\end{aligned} \quad (56)$$

up to the higher-order corrections beyond the present accuracy. Substituting these results into (32), we obtain the final form for the relevant twist-3 contribution to the hadronic tensor in SIDIS, as the factorization formula in terms of the gauge-invariant three-gluon correlation functions,

$$w(p,q,p_c) = \int\frac{dx_1}{x_1}\int\frac{dx_2}{x_2}\left.\frac{\partial S_{\mu\nu\lambda}^{abc}(k_1,k_2,q,p_c)p^{\lambda}}{\partial k_2^{\sigma}}\right|_{k_i=x_ip}\omega^{\mu}{}_{\alpha}\,\omega^{\nu}{}_{\beta}\,\omega^{\sigma}{}_{\gamma}\,\mathcal{M}_{F,abc}^{\alpha\beta\gamma}(x_1,x_2), \quad (57)$$

where we have restored the color indices a, b, c as well as momentum variables q, p_c, which were associated with the hard-scattering function in (32), and $\mathcal{M}_{F,abc}^{\alpha\beta\gamma}(x_1, x_2)$ denote the three-gluon lightcone correlation functions, obtained by integrating (56) over $k_i^-, \mathbf{k}_{i\perp}$ ($i = 1, 2$), as

$$\begin{aligned}\mathcal{M}_{F,abc}^{\alpha\beta\gamma}(x_1, x_2) &= -gi^3 \int \frac{d\lambda}{2\pi} \int \frac{d\mu}{2\pi} e^{i\lambda x_1} e^{i\mu(x_2 - x_1)} \langle pS | F_b^{\beta n}(0) F_c^{\gamma n}(\mu n) F_a^{\alpha n}(\lambda n) | pS \rangle \\ &= \frac{3}{40} d^{abc} O^{\alpha\beta\gamma}(x_1, x_2) - \frac{i}{24} f^{abc} N^{\alpha\beta\gamma}(x_1, x_2), \end{aligned} \tag{58}$$

with $O^{\alpha\beta\gamma}(x_1, x_2)$ and $N^{\alpha\beta\gamma}(x_1, x_2)$ in (1) and (2). As we demonstrated above in deriving these results, all the gauge-noninvariant terms that could potentially contribute to $w(p, q, p_c)$ vanished or canceled among themselves, and the *total twist-3* contribution to $w(p, q, p_c)$, relevant to SSA, proves to be expressed solely in terms of the gauge-invariant three-gluon correlation functions $O(x_1, x_2)$ and $N(x_1, x_2)$ defined as (7) and (8).

When one calculates the relevant hard part, $\partial S_{\mu\nu\lambda}^{abc}(k_1, k_2, q, p_c) p^\lambda / \partial k_2^\sigma \big|_{k_i = x_i p}$, arising in (57), one should note that the derivative with respect to k_2^σ can hit the delta functions, $\delta\left((k_2 + q - p_c)^2 - m_c^2\right)$ and $\delta\left((p_c + k_1 - k_2)^2 - m_c^2\right)$, which are involved in $S_{\mu\nu\lambda}^{abc}(k_1, k_2, q, p_c) p^\lambda$ as mentioned above (39). Such derivatives of these delta functions can be reexpressed by the derivatives with respect to x_2, and then be treated by integration by parts, giving rise to the derivative of the three-gluon correlation functions of (7) and (8). After such manipulations, the former of the above delta functions, which represents the on-shell condition for the final-state cut on the unobserved $\bar{c}$-quark line in the diagrams of Fig. 2, becomes

$$\delta\left((k_2 + q - p_c)^2 - m_c^2\right)\Big|_{k_2 = xp} = \frac{1}{\hat{z} Q^2} \delta\left(\frac{q_T^2}{Q^2} - \left(\frac{1}{\hat{x}} - 1\right)\left(\frac{1}{\hat{z}} - 1\right) + \frac{m_c^2}{\hat{z}^2 Q^2}\right), \tag{59}$$

with $\hat{x} = \frac{x_{bj}}{x}$, and this factor appears in the final expression for our cross section based on (57).

Here, we make a brief comment on the calculation presented in [32]. Using the notation in the present paper, the authors of [32] calculated $w\left(p, q, p_c\right)$ with the following formula, in place of the right-hand side of (57):

$$\int \frac{dx_1}{x_1} \int \frac{dx_2}{x_2} \left. \frac{\partial S_{\mu\nu\lambda}^{abc}(k_1, k_2, q, p_c) p^\lambda g_\perp^{\mu\nu}}{\partial k_{2\perp}^\sigma} \right|_{k_i = x_i p} \omega^\sigma{}_\gamma \, g_{\alpha\beta} \mathcal{M}_{F,abc}^{\alpha\beta\gamma}(x_1, x_2), \tag{60}$$

where

$$g_\perp^{\mu\nu} = g^{\mu\nu} - p^\mu n^\nu - p^\nu n^\mu = -S^\mu S^\nu - \epsilon^{\mu p n S} \epsilon^{\nu p n S}. \tag{61}$$

In (60), we can make the replacement $\omega^\sigma{}_\gamma \to g_{\perp\gamma}^\sigma$, up to the irrelevant corrections of twist-4 and higher, and, using (61) and the property $S_\gamma g_{\alpha\beta} \mathcal{M}_{F,abc}^{\alpha\beta\gamma}(x_1, x_2) = 0$, implied by (58) with (7) and (8), we see that the three-gluon correlation functions involved in (60) are indeed expressed by the two types of functions of (11) after evaluating the SGP at $x_1 = x_2$ explicitly.

Clearly, (60) used in [32] leads to a result different from the result based on our complete formula (57). It is straightforward to see that, if the tensor structure of the three-gluon correlation functions $\mathcal{M}^{\alpha\beta\gamma}_{F,abc}(x_1, x_2)$ of (58) were assumed to be given by only one structure, $g^{\alpha\beta}\epsilon^{\gamma pnS}$, our formula (57) would reduce to the formula (60), up to the corrections of twist-4 and higher. However, such assumption contradicts with the permutation symmetry required by the Bose statistics of the gluon, as emphasized in section 2 and represented in (7) and (8).

3.3 Calculation of $L_{\mu\nu}W^{\mu\nu}$

Using the kinematic variables defined in section 3.1, the differential cross section (28) can be expressed as

$$\frac{d^5\Delta\sigma}{dx_{bj}dQ^2dz_fdq_T^2d\phi} = \frac{\alpha_{em}^2}{64\pi^3x_{bj}^2S_{ep}^2Q^2}z_fL^{\mu\nu}(\ell,\ell')W_{\mu\nu}(p,q,P_h), \tag{62}$$

where $\alpha_{em} = e^2/(4\pi)$ is the QED coupling constant. We restore the implicit "free" Lorentz indices in (57) for the virtual photon, corresponding to μ and ν in $w_{\mu\nu}(p, q, p_c)$ of (31) (see the discussion above (32)), and, substituting the result into (31), we obtain the three-gluon-correlation contribution to $W_{\mu\nu}(p, q, P_h)$ in (62). To calculate the contraction $L^{\mu\nu}(\ell, \ell')W_{\mu\nu}(p, q, P_h)$ arising in (62), we introduce the following four vectors which are orthogonal to each other:

$$\begin{aligned}
T^\mu &= \frac{1}{Q}\left(q^\mu + 2x_{bj}p^\mu\right),\\
X^\mu &= \frac{1}{q_T}\left\{\frac{P_h^\mu}{z_f} - q^\mu - \left(1 + \frac{q_T^2 + m_h^2/z_f^2}{Q^2}\right)x_{bj}p^\mu\right\},\\
Y^\mu &= \epsilon^{\mu\nu\rho\sigma}Z_\nu X_\rho T_\sigma,\\
Z^\mu &= -\frac{q^\mu}{Q}.
\end{aligned} \tag{63}$$

These are the extension of four basis vectors introduced for the massless case ($m_h = 0$) in [48] to the case of massive meson with m_h in the final state [32]. These vectors become $T^\mu = (1, 0, 0, 0)$, $X^\mu = (0, 1, 0, 0)$, $Y^\mu = (0, 0, 1, 0)$, $Z^\mu = (0, 0, 0, 1)$ in the hadron frame. In the present case, we find that $W^{\mu\nu}$ can be expanded in terms of the following six independent tensors [29]:

$$\begin{aligned}
\mathcal{V}_1^{\mu\nu} &= X^\mu X^\nu + Y^\mu Y^\nu, & \mathcal{V}_2^{\mu\nu} &= g^{\mu\nu} + Z^\mu Z^\nu,\\
\mathcal{V}_3^{\mu\nu} &= T^\mu X^\nu + X^\mu T^\nu, & \mathcal{V}_4^{\mu\nu} &= X^\mu X^\nu - Y^\mu Y^\nu,\\
\mathcal{V}_8^{\mu\nu} &= T^\mu Y^\nu + Y^\mu T^\nu, & \mathcal{V}_9^{\mu\nu} &= X^\mu Y^\nu + Y^\mu X^\nu,
\end{aligned} \tag{64}$$

where we have followed the notation in [48] but have not shown the explicit form of the three tensors $\mathcal{V}_{5,6,7}$ among nine basis tensors because these three tensors are irrelevant for the expansion of our $W^{\mu\nu}$. We also introduce the inverse tensors $\widetilde{\mathcal{V}}_k^{\mu\nu}$ for the above $\mathcal{V}_k^{\mu\nu}$:

$$\widetilde{\mathcal{V}}_1^{\mu\nu} = \frac{1}{2}(2T^\mu T^\nu + X^\mu X^\nu + Y^\mu Y^\nu), \qquad \widetilde{\mathcal{V}}_2^{\mu\nu} = T^\mu T^\nu,$$

$$\widetilde{\mathcal{V}}_3^{\mu\nu} = -\frac{1}{2}(T^\mu X^\nu + X^\mu T^\nu), \qquad \widetilde{\mathcal{V}}_4^{\mu\nu} = \frac{1}{2}(X^\mu X^\nu - Y^\mu Y^\nu),$$

$$\widetilde{\mathcal{V}}_8^{\mu\nu} = \frac{-1}{2}(T^\mu Y^\nu + Y^\mu T^\nu), \qquad \widetilde{\mathcal{V}}_9^{\mu\nu} = \frac{1}{2}(X^\mu Y^\nu + Y^\mu X^\nu). \tag{65}$$

Then, one obtains

$$L_{\mu\nu} W^{\mu\nu} = \sum_{k=1,\cdots,4,8,9} \left[L_{\mu\nu}\mathcal{V}_k^{\mu\nu}\right]\left[W_{\rho\sigma}\widetilde{\mathcal{V}}_k^{\rho\sigma}\right] \equiv Q^2 \sum_{k=1,\cdots,4,8,9} \mathcal{A}_k \left[W_{\rho\sigma}\widetilde{\mathcal{V}}_k^{\rho\sigma}\right], \tag{66}$$

where $\mathcal{A}_k \equiv L_{\mu\nu}\mathcal{V}_k^{\mu\nu}/Q^2$ is given by

$$\begin{aligned}
\mathcal{A}_1 &= 1 + \cosh^2\psi, \\
\mathcal{A}_2 &= -2, \\
\mathcal{A}_3 &= -\cos\phi \sinh 2\psi, \\
\mathcal{A}_4 &= \cos 2\phi \sinh^2\psi, \\
\mathcal{A}_8 &= -\sin\phi \sinh 2\psi, \\
\mathcal{A}_9 &= \sin 2\phi \sinh^2\psi.
\end{aligned} \tag{67}$$

By the expansion (66), the cross section for $ep^\uparrow \to eDX$ consists of the five structure functions associated with $\mathcal{A}_{1,2}$, $\mathcal{A}_3$, $\mathcal{A}_4$, $\mathcal{A}_8$ and $\mathcal{A}_9$, respectively, which have different dependences on the azimuthal angle ϕ.

In closing this section, we summarize the prescription established for calculating the twist-3 single-spin-dependent cross section that is generated by the three-gluon correlation functions of the nucleon: The corresponding differential cross section is given by (62) with the expansion in the right-hand side of (66), in which $W_{\rho\sigma}$ is given as (31) using our factorization formula (57) for $w_{\mu\nu}(p, q, p_c)$.

4 Result for the twist-3 cross section for $ep^\uparrow \to eDX$

Using the formalism presented above, we now obtain the leading-order QCD formula for the single-spin-dependent cross section in the SIDIS, $ep^\uparrow \to eDX$, generated from the twist-3

three-gluon correlation functions $O(x_1,x_2)$ and $N(x_1,x_2)$ of (7) and (8), as

$$\frac{d^5\Delta\sigma}{dx_{bj}dQ^2dz_fdq_T^2d\phi}$$

$$= \frac{\alpha_{em}^2\alpha_s e_c^2 M_N}{8\pi x_{bj}^2 S_{ep}^2 Q^2}\left(\frac{-\pi}{2}\right)\sum_{k=1,\cdots,4,8,9}\mathcal{A}_k\mathcal{S}_k\int_{x_{\min}}^1\frac{dx}{x}\int_{z_{\min}}^1\frac{dz}{z}\delta\left(\frac{q_T^2}{Q^2}-\left(1-\frac{1}{\hat{x}}\right)\left(1-\frac{1}{\hat{z}}\right)+\frac{m_c^2}{\hat{z}^2Q^2}\right)$$

$$\times\sum_{a=c,\bar{c}}D_a(z)\left[\delta_a\left\{\left(\frac{d}{dx}O(x,x)-\frac{2O(x,x)}{x}\right)\Delta\hat{\sigma}_k^1+\left(\frac{d}{dx}O(x,0)-\frac{2O(x,0)}{x}\right)\Delta\hat{\sigma}_k^2\right.\right.$$

$$\left.+\frac{O(x,x)}{x}\Delta\hat{\sigma}_k^3+\frac{O(x,0)}{x}\Delta\hat{\sigma}_k^4\right\}$$

$$+\left\{\left(\frac{d}{dx}N(x,x)-\frac{2N(x,x)}{x}\right)\Delta\hat{\sigma}_k^1-\left(\frac{d}{dx}N(x,0)-\frac{2N(x,0)}{x}\right)\Delta\hat{\sigma}_k^2\right.$$

$$\left.\left.+\frac{N(x,x)}{x}\Delta\hat{\sigma}_k^3-\frac{N(x,0)}{x}\Delta\hat{\sigma}_k^4\right\}\right], \tag{68}$$

where the subscript k runs over $1,2,3,4,8,9$ with $\mathcal{A}_k$ defined in (67) and $\mathcal{S}_k$ defined as $\mathcal{S}_k=\sin\Phi_S$ for $k=1,2,3,4$ and $\mathcal{S}_k=\cos\Phi_S$ for $k=8,9$. The quark-flavor index a can, in principle, be c and $\bar{c}$, with $\delta_c=1$ and $\delta_{\bar{c}}=-1$, so that the cross section for the $\bar{D}$-meson production $ep^\uparrow\to e\bar{D}X$ can be obtained by a simple replacement of the fragmentation function to that for the $\bar{D}$ meson, $D_a(z)\to\bar{D}_a(z)$. $\alpha_s=g^2/(4\pi)$ is the strong coupling constant, and $e_c=2/3$ represents the electric charge of the c-quark. The lower limits of the integrals are given by

$$z_{\min}=z_f\frac{(1-x_{bj})\,Q^2}{2x_{bj}m_c^2}\left(1-\sqrt{1-\frac{4x_{bj}m_c^2}{(1-x_{bj})\,Q^2}\left[1+\frac{x_{bj}q_T^2}{(1-x_{bj})\,Q^2}\right]}\right), \tag{69}$$

and

$$x_{\min}=\begin{cases} x_{bj}\left[1+\frac{z_f^2q_T^2+m_c^2}{z_f(1-z_f)Q^2}\right] & \text{for}\quad z_f\left(1+\sqrt{1+\frac{q_T^2}{m_c^2}}\right)>1,\\ x_{bj}\left[1+\frac{2m_c^2}{Q^2}\left(1+\sqrt{1+\frac{q_T^2}{m_c^2}}\right)\right] & \text{for}\quad z_f\left(1+\sqrt{1+\frac{q_T^2}{m_c^2}}\right)\le 1.\end{cases} \tag{70}$$

Partonic hard cross sections $\Delta\hat{\sigma}_k^i$ $(i = 1, \cdots, 4)$ are obtained as follows:

$$\begin{cases} \Delta\hat{\sigma}_1^1 = \frac{8q_T\hat{x}}{Q^6(1-\hat{z})^3\hat{z}^2}\left\{Q^4\hat{z}(1-\hat{z})(1-2\hat{z}+2\hat{z}^2-2\hat{x}+2\hat{x}^2+12\hat{x}\hat{z}(1-\hat{x})(1-\hat{z}))\right. \\ \qquad\qquad \left.+2m_c^2Q^2\hat{x}(2\hat{z}(1-\hat{z})+\hat{x}(1-8\hat{z}+8\hat{z}^2))-4m_c^4\hat{x}^2\right\}, \\ \Delta\hat{\sigma}_2^1 = \frac{64q_T\hat{x}^2}{Q^4(1-\hat{z})^2\hat{z}}\left\{Q^2\hat{z}(1-\hat{x})(1-\hat{z})-m_c^2\hat{x}\right\}, \\ \Delta\hat{\sigma}_3^1 = \frac{16\hat{x}}{Q^5(1-\hat{z})^3\hat{z}^3}(1-2\hat{z})\left\{Q^2\hat{z}(1-\hat{x})(1-\hat{z})-m_c^2\hat{x}\right\}\left\{Q^2\hat{z}(1-2\hat{x})(1-\hat{z})-2m_c^2\hat{x}\right\}, \\ \Delta\hat{\sigma}_4^1 = \frac{32q_T\hat{x}^2}{Q^6(1-\hat{z})^3\hat{z}^2}\left\{Q^2\hat{z}(1-\hat{x})(1-\hat{z})-m_c^2\hat{x}\right\}\left\{Q^2\hat{z}(1-\hat{z})+m_c^2\right\}, \\ \Delta\hat{\sigma}_8^1 = \Delta\hat{\sigma}_9^1 = 0, \end{cases} \tag{71}$$

$$\begin{cases} \Delta\hat{\sigma}_1^2 = \frac{8q_T\hat{x}}{Q^6(1-\hat{z})^3\hat{z}^2}\{Q^4\hat{z}(1-\hat{z})(1-2\hat{z}+2\hat{z}^2-4\hat{x}+4\hat{x}^2+24\hat{x}\hat{z}(1-\hat{x})(1-\hat{z})) \\ \qquad\qquad +4m_c^2Q^2\hat{x}(2\hat{z}(1-\hat{z})+\hat{x}(1-8\hat{z}+8\hat{z}^2))-8m_c^4\hat{x}^2\}, \\ \Delta\hat{\sigma}_2^2 = 2\Delta\hat{\sigma}_2^1, \\ \Delta\hat{\sigma}_3^2 = 2\Delta\hat{\sigma}_3^1, \\ \Delta\hat{\sigma}_4^2 = -\frac{16q_T\hat{x}}{Q^6(1-\hat{z})^3\hat{z}^2}(Q^2\hat{z}(1-2\hat{x})(1-\hat{z})-2m_c^2\hat{x})^2, \\ \Delta\hat{\sigma}_8^2 = \frac{16\hat{x}}{Q^3(1-\hat{z})^2\hat{z}^2}(1-2\hat{z})(Q^2\hat{z}(1-\hat{x})(1-\hat{z})-m_c^2\hat{x}), \\ \Delta\hat{\sigma}_9^2 = -\frac{16q_T\hat{x}}{Q^4(1-\hat{z})^2\hat{z}}(Q^2\hat{z}(1-2\hat{x})(1-\hat{z})-2m_c^2\hat{x}), \end{cases} \tag{72}$$

$$\begin{cases} \Delta\hat{\sigma}_1^3 = \frac{16q_T\hat{x}^2}{Q^6(1-\hat{z})^3\hat{z}^2}(Q^2\hat{z}(1-2\hat{x})(1-\hat{z})-2m_c^2\hat{x})(Q^2(1-6\hat{z}+6\hat{z}^2)-2m_c^2), \\ \Delta\hat{\sigma}_2^3 = -\frac{64q_T\hat{x}^2}{Q^4(1-\hat{z})^2\hat{z}}(Q^2\hat{z}(1-2\hat{x})(1-\hat{z})-2m_c^2\hat{x}), \\ \Delta\hat{\sigma}_3^3 = -\frac{8\hat{x}}{Q^5(1-\hat{z})^3\hat{z}^3}(1-2\hat{z})\left\{Q^4\hat{z}^2(1-\hat{z})^2(1-8\hat{x}+8\hat{x}^2)\right. \\ \qquad\qquad\qquad \left.-8m_c^2Q^2\hat{x}\hat{z}(1-2\hat{x})(1-\hat{z})+8m_c^4\hat{x}^2\right\}, \\ \Delta\hat{\sigma}_4^3 = -\frac{32q_T\hat{x}^2}{Q^6(1-\hat{z})^3\hat{z}^2}(Q^2\hat{z}(1-\hat{z})+m_c^2)(Q^2\hat{z}(1-2\hat{x})(1-\hat{z})-2m_c^2\hat{x}), \\ \Delta\hat{\sigma}_8^3 = -\frac{8\hat{x}}{Q^3(1-\hat{z})^2\hat{z}^2}(1-2\hat{z})(Q^2\hat{z}(1-2\hat{x})(1-\hat{z})-2m_c^2\hat{x}), \\ \Delta\hat{\sigma}_9^3 = -\frac{32q_T\hat{x}^2}{Q^4(1-\hat{z})^2\hat{z}}(Q^2\hat{z}(1-\hat{z})+m_c^2), \end{cases} \tag{73}$$

$$
\begin{cases}
\Delta\hat{\sigma}_1^4 = \frac{16q_T\hat{x}^2}{Q^6(1-\hat{z})^3\hat{z}^2}(Q^2\hat{z}(1-4\hat{x})(1-\hat{z})-4m_c^2\hat{x})(Q^2(1-6\hat{z}+6\hat{z}^2)-2m_c^2), \\
\Delta\hat{\sigma}_2^4 = -\frac{64q_T\hat{x}^2}{Q^4(1-\hat{z})^2\hat{z}}(Q^2\hat{z}(1-4\hat{x})(1-\hat{z})-4m_c^2\hat{x}), \\
\Delta\hat{\sigma}_3^4 = \frac{8\hat{x}}{Q^5(1-\hat{z})^3\hat{z}^3}(1-2\hat{z})\left\{Q^4\hat{z}^2(1-\hat{z})^2(1+12\hat{x}-16\hat{x}^2)\right. \\
\qquad\qquad \left.+4m_c^2Q^2\hat{x}\hat{z}(3-8\hat{x})(1-\hat{z})-16m_c^4\hat{x}^2\right\}, \\
\Delta\hat{\sigma}_4^4 = -\frac{32q_T\hat{x}}{Q^6(1-\hat{z})^3\hat{z}^2}(Q^4\hat{z}^2(1-\hat{z})^2(1+\hat{x}-4\hat{x}^2)+m_c^2Q^2\hat{x}\hat{z}(1-8\hat{x})(1-\hat{z})-4m_c^4\hat{x}^2), \\
\Delta\hat{\sigma}_8^4 = \frac{8\hat{x}}{Q^3(1-\hat{z})^2\hat{z}^2}(1-2\hat{z})(Q^2\hat{z}(1+2\hat{x})(1-\hat{z})+2m_c^2\hat{x}), \\
\Delta\hat{\sigma}_9^4 = -\frac{32q_T\hat{x}}{Q^4(1-\hat{z})^2\hat{z}}(Q^2\hat{z}(1-\hat{z})(1+\hat{x})+m_c^2\hat{x}),
\end{cases}
\tag{74}
$$

where

$$
\widehat{x} = \frac{x_{bj}}{x}, \qquad \widehat{z} = \frac{z_f}{z}. \tag{75}
$$

The single-spin-dependent cross section (68) can be decomposed into the five structure functions, based on the different dependences on the azimuthal angles Φ_S and ϕ through the above-mentioned explicit forms of $\mathcal{A}_k$ and $\mathcal{S}_k$, as

$$
\frac{d^5\Delta\sigma}{dx_{bj}dQ^2dz_fdq_T^2d\phi} = \sin\Phi_S\left(\mathcal{F}_1+\mathcal{F}_2\cos\phi+\mathcal{F}_3\cos 2\phi\right)
$$
$$
+\cos\Phi_S\left(\mathcal{F}_4\sin\phi+\mathcal{F}_5\sin 2\phi\right). \tag{76}
$$

The five independent azimuthal structures of this type have been observed also in the twist-3 single-spin-dependent cross section for $ep^\uparrow \to e\pi X$, generated from the quark-gluon correlation functions, as presented in [29, 35, 36]. Introducing the azimuthal angles ϕ_h and ϕ_S of the hadron plane and the nucleon's spin vector $\vec{S}$, respectively, as measured from the *lepton plane*, they are connected to the above Φ_S and ϕ as $\Phi_S = \phi_h - \phi_S$, $\phi = \phi_h$. One can recast (76) into the superposition of five sine modulations with these new azimuthal angles as

$$
\frac{d^5\Delta\sigma}{dx_{bj}dQ^2dz_fdq_T^2d\phi_h} = \sin(\phi_h-\phi_S)\,F^{\sin(\phi_h-\phi_S)}+\sin(2\phi_h-\phi_S)\,F^{\sin(2\phi_h-\phi_S)}+\sin\phi_S\,F^{\sin\phi_S}
$$
$$
+\sin(3\phi_h-\phi_S)\,F^{\sin(3\phi_h-\phi_S)}+\sin(\phi_h+\phi_S)\,F^{\sin(\phi_h+\phi_S)}, \tag{77}
$$

with the relations:

$$
F^{\sin(\phi_h-\phi_S)} = \mathcal{F}_1, \quad F^{\sin(2\phi_h-\phi_S)} = \frac{\mathcal{F}_2+\mathcal{F}_4}{2}, \quad F^{\sin\phi_S} = \frac{-\mathcal{F}_2+\mathcal{F}_4}{2},
$$
$$
F^{\sin(3\phi_h-\phi_S)} = \frac{\mathcal{F}_3+\mathcal{F}_5}{2}, \quad F^{\sin(\phi_h+\phi_S)} = \frac{-\mathcal{F}_3+\mathcal{F}_5}{2}. \tag{78}
$$

The azimuthal-angle dependence of the single-spin-dependent cross section derived in the TMD approach [18] was presented in a form similar to (77), so that the decomposition (77) into five structure functions is convenient to make connection with the TMD approach in the small-q_T region.

For completeness, we consider the unpolarized cross section for the SIDIS, $ep \to eDX$, and list the corresponding twist-2 contribution at the leading order in QCD perturbation theory, which gives an extension of the study in [49, 48, 50] to the case with massive-hadron production in the final state. The result is

$$\frac{d^5\sigma^{\rm unpol}}{dx_{bj}dQ^2dz_f dq_T^2 d\phi} = \frac{\alpha_{em}^2\alpha_s e_c^2}{8\pi x_{bj}^2 S_{ep}^2 Q^2}\frac{1}{4}\sum_{k=1}^{4}\mathcal{A}_k\int_{x_{\min}}^1\frac{dx}{x}\int_{z_{\min}}^1\frac{dz}{z}\sum_{a=c,\bar{c}}D_a(z)G(x)\hat{\sigma}_k^U$$
$$\times\delta\left(\frac{q_T^2}{Q^2}-\left(1-\frac{1}{\hat{x}}\right)\left(1-\frac{1}{\hat{z}}\right)+\frac{m_c^2}{\hat{z}^2Q^2}\right), \tag{79}$$

with (67), (69), and (70). This is generated from the unpolarized gluon-density distribution $G(x)$ for the nucleon,

$$G(x) = -\frac{g_{\beta\alpha}}{x}\int\frac{d\lambda}{2\pi}e^{i\lambda x}\langle p|F_a^{\beta n}(0)F_a^{\alpha n}(\lambda n)|p\rangle, \tag{80}$$

and the partonic hard cross sections are obtained as, utilizing the formalism discussed in section 3.3,

$$\begin{cases} \hat{\sigma}_1^U &= \frac{4}{Q^4(1-\hat{z})^2\hat{z}^2}\{Q^4\hat{z}(1-\hat{z})(1-2\hat{z}+2\hat{z}^2-2\hat{x}+2\hat{x}^2+12\hat{x}\hat{z}(1-\hat{x})(1-\hat{z})) \\ & \quad +2m_c^2Q^2\hat{x}(2\hat{z}(1-\hat{z})+\hat{x}(1-8\hat{z}+8\hat{z}^2))-4m_c^4\hat{x}^2\}, \\ \hat{\sigma}_2^U &= \frac{32\hat{x}}{Q^2(1-\hat{z})\hat{z}}(Q^2\hat{z}(1-\hat{x})(1-\hat{z})-m_c^2\hat{x}), \\ \hat{\sigma}_3^U &= \frac{8q_T\hat{x}}{Q^3(1-\hat{z})^2\hat{z}}(1-2\hat{z})(Q^2\hat{z}(1-2\hat{x})(1-\hat{z})-2m_c^2\hat{x}), \\ \hat{\sigma}_4^U &= \frac{16\hat{x}}{Q^4(1-\hat{z})^2\hat{z}^2}(Q^2\hat{z}(1-\hat{x})(1-\hat{z})-m_c^2\hat{x})(Q^2\hat{z}(1-\hat{z})+m_c^2). \end{cases} \tag{81}$$

We note that these partonic hard cross sections coincide with the corresponding results presented in [32], except for $\hat{\sigma}_1^U$. Comparing (81) for $\hat{\sigma}_k^U$ with the above result (71) for $\Delta\hat{\sigma}_k^1$, which represents the partonic hard cross sections associated with the derivatives, $dO(x,x)/dx$ and $dN(x,x)/dx$, of the three-gluon correlation functions in (68), one finds the following relations between the partonic hard cross sections at the twist-3 level and those at the twist-2 level:

$$\Delta\hat{\sigma}_k^1 = \frac{2q_T\widehat{x}}{Q^2(1-\widehat{z})}\hat{\sigma}_k^U. \tag{82}$$

The other partonic cross sections at the twist-3 level, $\Delta\hat{\sigma}_k^i$ ($i=2,3,4$) of (72)-(74), are also related to the partonic cross sections (81) at the twist-2 level, although, unlike (82), the corresponding relations cannot be manifested by direct comparison between the formulae of

(72)-(74) and those of (81). These remarkable relations, as well as a single "master formula" behind them, will be presented elsewhere [51]. We mention that the similar master formula, which allows us to relate the $3 \to 2$ partonic subprocess relevant for the twist-3 level to the $2 \to 2$ subprocess for the twist-2 level, was derived [29] for the case of the SGP contributions associated with the twist-3 quark-gluon correlation functions.

5 Summary

In this paper, we have investigated the single spin asymmetry for the D-meson production in SIDIS, generated from the twist-3 three-gluon correlation functions for the nucleon. We first showed, correcting the previous study in [24], that there are only two independent three-gluon correlation functions of twist-3, $O(x_1, x_2)$ and $N(x_1, x_2)$, which correspond to two possible ways to construct color-singlet combination composed of three active gluons. Then, we have formulated the method for calculating the twist-3 single-spin-dependent cross section generated from the three-gluon correlations. Our formulation is based on a systematic analysis of the relevant diagrams in the Feynman gauge and gives all the contribution to the cross section at the twist-3 level in the leading order in perturbative QCD, guaranteeing the gauge invariance of the result. As in the twist-3 mechanism for SSA generated from the quark-gluon correlation functions for the nucleon, the cross section in the present case occurs as the pole contribution of an internal propagator in the partonic hard-scattering subprocess, and the corresponding contribution leads to the cross section expressed in terms of the four types of functions of the relevant momentum fraction x: $O(x, x)$, $O(x, 0)$, $N(x, x)$ and $N(x, 0)$. We find that all these four-types of functions and their derivatives with respect to x contribute to the final form of the cross section. These features discussed for the SSA in $ep^{\uparrow} \to eDX$ also apply to the case of A_N in $p^{\uparrow}p \to hX$ ($h = \pi,\ K,\ D$, etc.) [52].

These new results have also revealed that the previous studies [32, 33] missed important contributions. In particular, the factorization formula for the cross section used in [32, 33] was written down in an ad-hoc way, and, compared to our factorization formula derived in this paper, involved additional projection for the Lorentz structure onto a particular component of the hard-scattering part as well as of the three-gluon correlation functions, which would have lead to the result incompatible with symmetry requirements in QCD.

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A Symmetry constraints on the gluon correlation functions

In this Appendix, we consider the correlation functions of the gluon fields, $A_a^\mu(\xi)$, in the nucleon. Such correlation functions of the two- or three-gluon fields, with non-lightlike separations between those fields, arise in the intermediate step of the analysis of the relevant Feynman diagrams for the SIDIS, $ep^\uparrow \to eDX$, to the twist-3 accuracy, as discussed in section 3.2. We discuss the symmetry properties of those correlation functions and their implication on SSA: We show that Fig. 1(a) does not contribute to SSA, and that Fig. 2(b) can give rise to SSA.

Similarly to (32), the contribution from Fig. 1(a) to $w_{\mu\nu}(p,q,p_c)$ in (31) can be written as,

$$\int \frac{d^4k}{(2\pi)^4} S_{\mu\nu}^{(2)}(k) M_{(2)}^{\mu\nu}(k,p,S), \tag{83}$$

where the Lorentz indices for the virtual photon in the partonic hard-scattering part $S_{\mu\nu}^{(2)}(k)$ are suppressed for simplicity, and the corresponding nucleon matrix element $M_{(2)}^{\mu\nu}(k,p,S)$ is defined as a correlation function of the type mentioned above:

$$M_{(2)}^{\mu\nu}(k,p,S) = \int d^4\xi e^{ik\cdot\xi} \langle pS|A_a^\nu(0)A_a^\mu(\xi)|pS\rangle. \tag{84}$$

Note that the color indices of the gluon fields in this formula are summed over, since only the color-singlet combination is relevant as a matrix element in the color-singlet hadron. Also, the corresponding color projection is taken for $S_{\mu\nu}^{(2)}(k)$ in (83). Now, invariance under the parity (P) and time-reversal (T) transformations implies

$$M_{(2)}^{\mu\nu}(k,p,S)^* = M_{(2)}^{\mu\nu}(k,p,-S), \tag{85}$$

and, combined with the fact that the matrix element $M_{(2)}^{\mu\nu}(k,p,S)$ depends on the spin vector S^μ *linearly*, we find that the spin-dependent part of $M_{(2)}^{\mu\nu}(k,p,S)$ is a pure imaginary quantity. For the process $ep^\uparrow \to eDX$, the leptonic tensor $L^{\mu\nu}$ of (28) is real. Accordingly, an imaginary contribution from the hard part $S_{\mu\nu}^{(2)}(k)$ is necessary to give the real contribution to the spin-dependent cross section. However, this is impossible for the leading-order diagrams contributing to the middle blob in Fig. 1(a). Therefore, (83) representing the contribution from Fig. 1(a) does not give rise to SSA.

Next, using the similar logic as above, we consider the contribution of Fig. 1(b) to (32). By projecting $M_{abc}^{\mu\nu\lambda}(k_1,k_2)$ in (33) into the color-singlet components, we define the two types of the contractions of the corresponding color indices as

$$\begin{aligned} M_\pm^{\mu\nu\lambda}(k_1,k_2,p,S) &\equiv C_\pm^{bca} M_{abc}^{\mu\nu\lambda}(k_1,k_2) \\ &= \int d\xi \int d\eta\, e^{ik_1\xi} e^{i(k_2-k_1)\eta} \langle pS|C_\pm^{bca} A_b^\nu(0) A_c^\lambda(\eta) A_a^\mu(\xi)|pS\rangle, \end{aligned} \tag{86}$$

with $C_\pm^{bca}$ of (12). PT-invariance implies

$$M_\pm^{\mu\nu\lambda}(k_1,k_2,p,S)^* = M_\pm^{\mu\nu\lambda}(k_1,k_2,p,-S), \tag{87}$$

which shows that the spin-dependent parts of $M_\pm^{\mu\nu\lambda}(k_1,k_2,p,S)$ are pure imaginary quantities. Accordingly, only an imaginary contribution from the corresponding color-projected hard part $C_\pm^{bca}S_{\mu\nu\lambda}^{abc}(k_1,k_2,q,p_c)$ can give rise to SSA with (32) in $ep^\uparrow \to eDX$. For the diagrams shown in Fig. 1(b), the pole contribution of an internal propagator in the hard part can give rise to such imaginary contribution. The corresponding unpinched poles are allowed in Fig. 1(b), owing to the presence of an extra gluon line connecting the hard and soft parts, compared with Fig. 1(a).

At this point one might recall that the $k_\perp$-dependent gluon distribution function [53] ("gluon-Sivers function") can give rise to SSA in the framework of the TMD factorization and this fact may contradict with the above statement that (84) does not contribute to SSA. However, the gluon Sivers function can become well-defined only after supplying an appropriate gauge-link operator between the gluon-fields, and the gauge-link operator actually resums the effect of the extra gluon-lines connecting the hard and soft parts, i.e., the gluon-Sivers function represents some effect of the contribution from Fig. 1(b). Therefore, there is no contradiction between the two facts. The gluon-Sivers function may rather represent the same effect as the three-gluon correlation functions in the region of the intermediate transverse momentum, similarly to the case of the quark-Sivers function and the twist-3 quark-gluon correlation functions as shown in [39, 40, 41].

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